

# Do CPS translations also translate realizers?

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CPS translations, compilation entre languages backtracking and control operators

Curry-Howard: traduction logique,  $\neg$

classical logic direct-style: control operators, Krivine realizability

# Realizability

Modèle interpretation formulas by set of programs

intuition for  $A \rightarrow B$

standard intuitionistic realizability is incompatible with classical logic

## Oliva-Streicher: Krivine is Kleene after Friedman

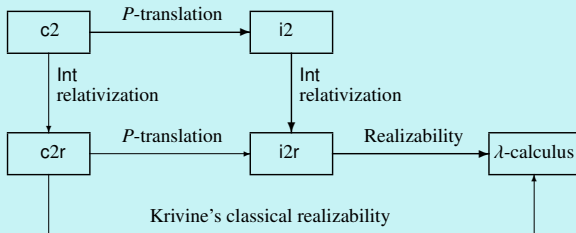


Figure 5: Alternative interpretation of **c2**

### This talk

Do  $t \Vdash A$  imply  $[t]_t \Vdash \llbracket A \rrbracket$ ?

# Krivine realizability

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# Realizability, a 3-steps recipe

❶ **formulas** (a.k.a. types)

↪ *simple types, 2<sup>nd</sup> – order logic, ZF, ...*

❷ a **computational system** (a.k.a. your favorite calculus)

↪ *some  $\lambda$  – calculus, a combinators algebra, PCF, etc.*

❸ formulas **interpretation**

# Realizability, a 3-steps recipe

next slide

❶ **formulas** (a.k.a. types)

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❷ a **computational system** (a.k.a. your favorite calculus)

↪ some  $\lambda$  – calculus, a combinators algebra, PCF, etc.

❸ **formulas interpretation** (a.k.a. truth values)

↪  $|A| = \{t \in \Lambda : t \Vdash A\}$



Definition etc...

# CPS translation

Sur un exemple ?

## Soundness

- 1 If  $\vdash t : A$  then  $\vdash [t]_t : \llbracket A \rrbracket$ .
- 2 If  $c \rightarrow c'$  then  $[c]_c \rightarrow [c']_c$ .

# Realizability interpretation

La structure de l'interprétation

pole  $\perp$  trois niveaux  $|A|_v, ||A||, |A|$

$t \Vdash A \rightarrow B$

## Adequacy

If  $\vdash t : A$  then  $t \Vdash A$ .

# Parenthesis

This defines an evidenced frame

(triplet + intuition ?)

la bonne notion pour dire “ceci est un modèle de réalisabilité”

(not shown: HA2 also induces an EF)

## CPS vs. realizers

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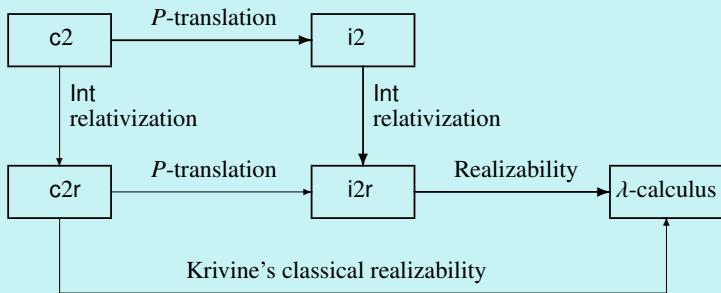
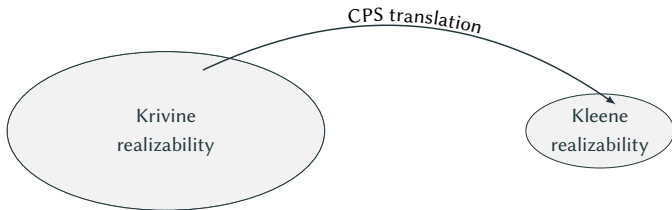


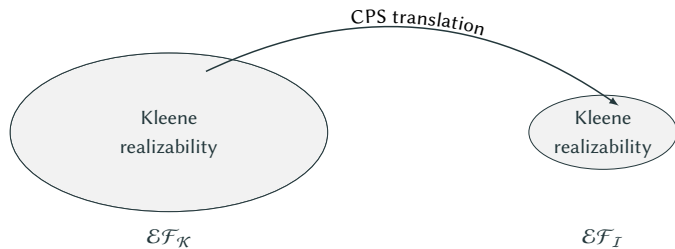
Figure 5: Alternative interpretation of c2



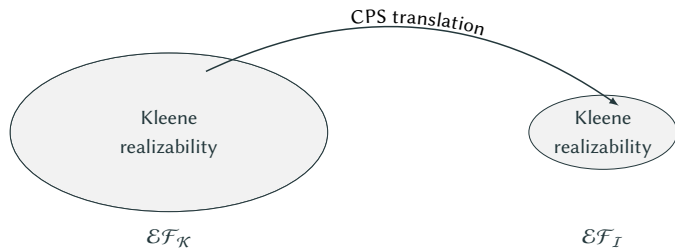
Parameters:

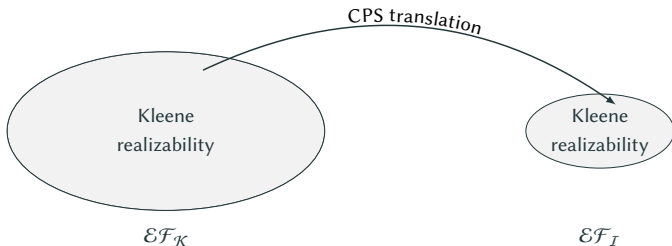
- pole in the source
- return type/pole in the target

“ $\neg$ ”  $A \simeq A \rightarrow \perp$

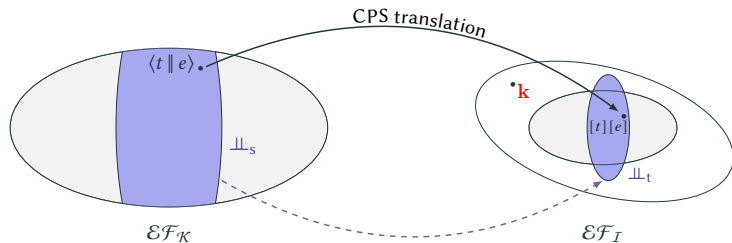








**Question** - Does the CPS define an EF morphism?



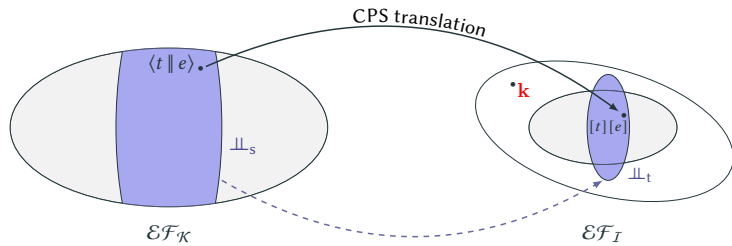
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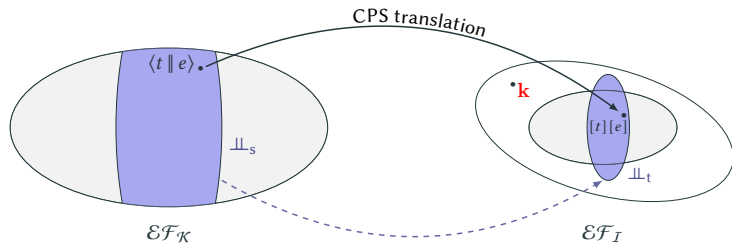
**Bad news**

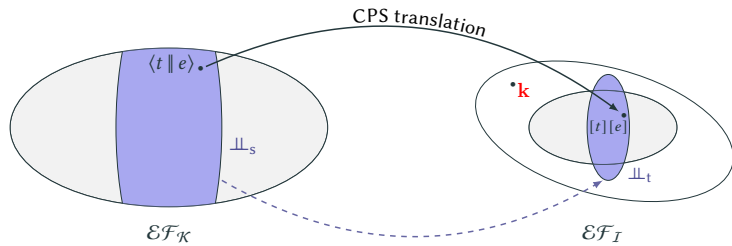
The CPS does not, in general, define an EF morphism.

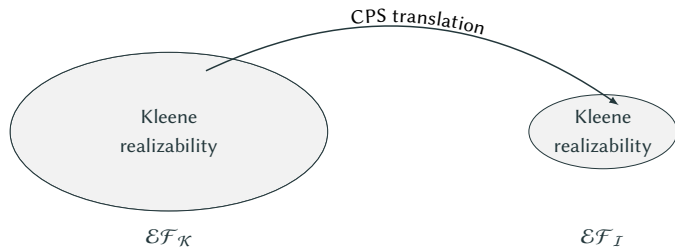
**Proof :**

We can exhibit  $t \Vdash_{\mathcal{K}} \mathbb{B}$  such that  $[t] \not\Vdash_{\mathcal{I}} (\mathbb{B} \rightarrow \mathcal{R}) \rightarrow \mathcal{R}$ .









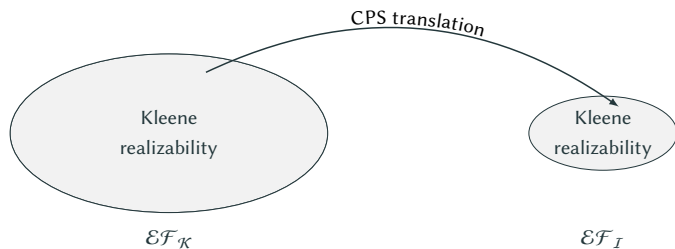
**Question** - Can we choose  $\perp_s$  and  $\perp_t = |\mathcal{R}|$  so that it works?

Oliva-Streicher:

$$t \Vdash_{\mathcal{K}} A \stackrel{\text{def}}{=} t \Vdash_{\mathcal{I}} \neg\neg A$$

Realizers = CPS-translated terms (or behaving like)

$\perp_t$  somehow traced backwards



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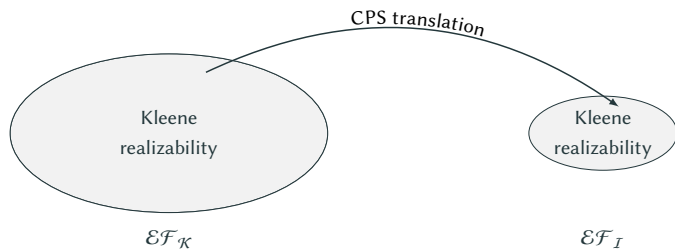
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### Proposition

❶ **Backward EF** - Given  $\perp_t$ , we can pick

$$\perp_c = \{c : [c] \in \perp_t\}$$





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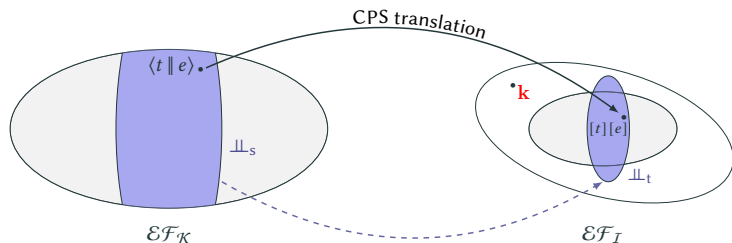
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## Proposition

② **Forward EF** - Given  $\perp_s$ , we can pick

$$\perp_t \triangleq \{t : \exists c \in \perp_s. t \rightarrow_R [c]\}$$



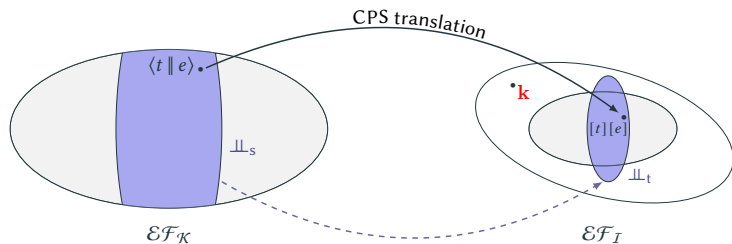
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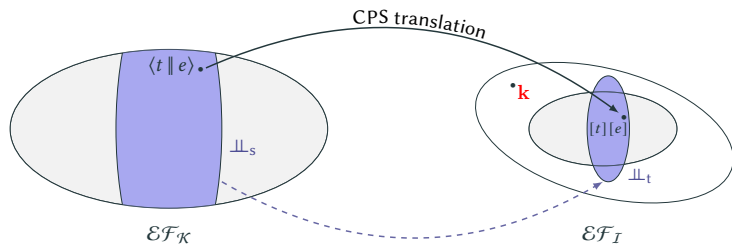
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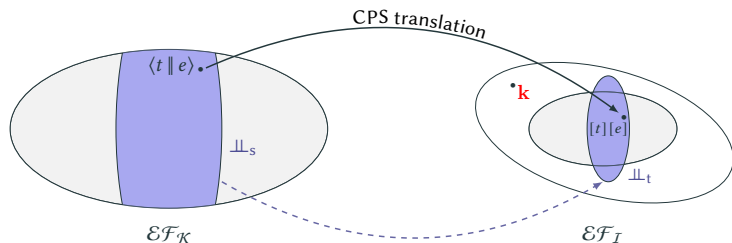
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## Conclusion

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## Open question

Are they realizers which are *always* compatible with CPS translations?

↪ *universal realizers?*

↪ *what about other syntactic translation/effects?*

Stronger notion of EF morphism preserving evidences ?

**The end**

*Thank you for your attention!*