Static Value Analysis by Abstract Interpretation for Functional Programs manipulating Recursive Algebraic Data Types

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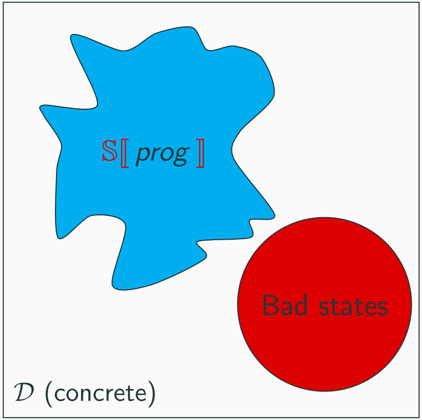
Introduction

Software Verification

Software bugs can be costly... and testing is not enough!

Abstract interpretation



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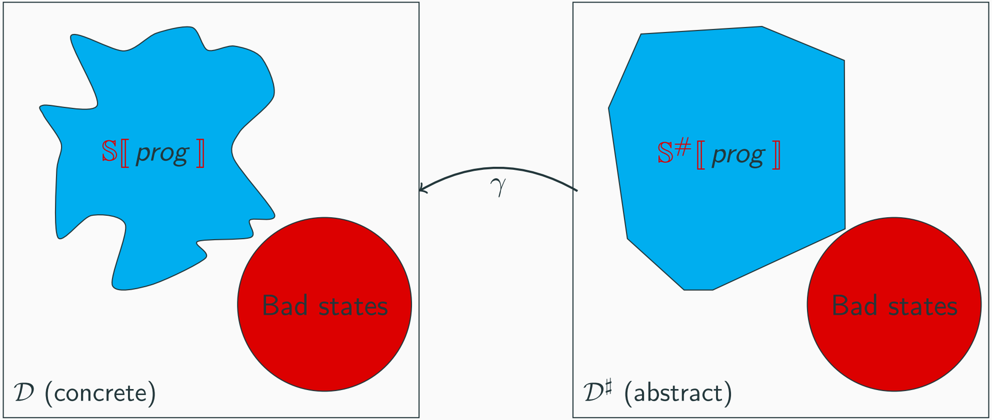
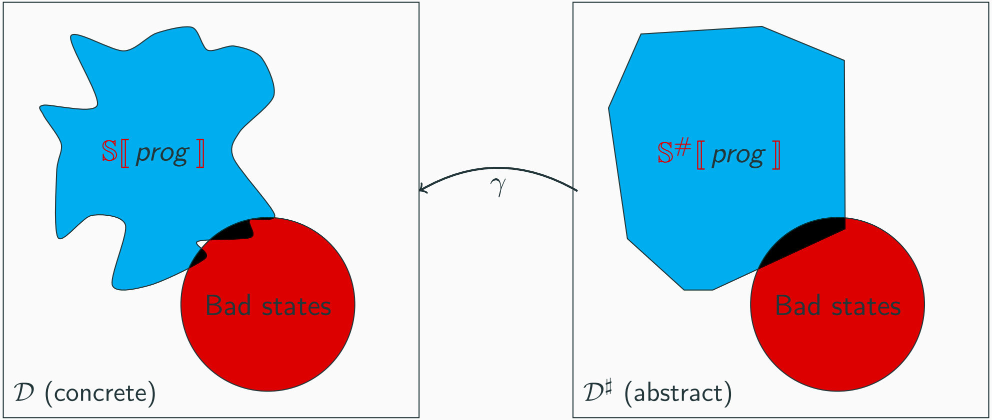


✓ Program correct

Abstract interpretation



✗ True alarm

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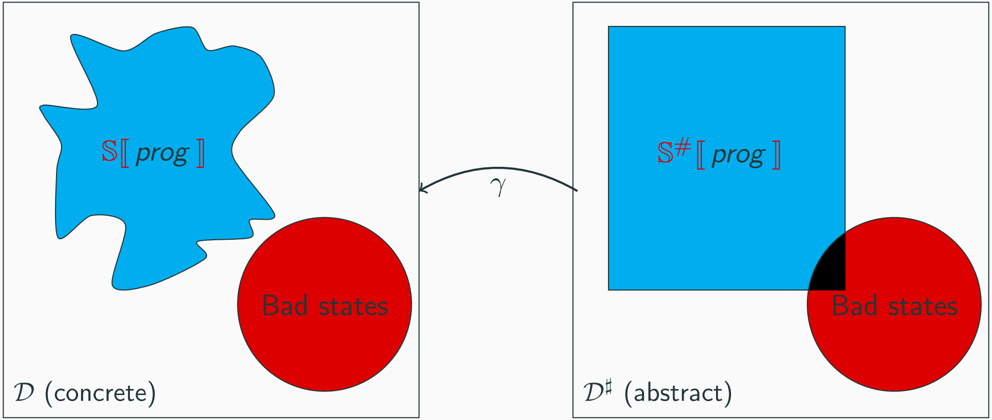

✗ False alarm (too unprecise)

Domains and relationality

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x = 0 ; y = 1 ;
while (y < 1000){
  if (rand(0,1) == 0) { x++; } else { x--; } ;
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- $y = 1000$, $-y < x < y$

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- ☐ Recursivity
- ☐ Algebraic Data Types
- ☐ Pattern-matching
- ☐ Higher Order
- ☐ Polymorphism

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- ✗ Polymorphism

Motivating example

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type list = Cons of int * list | Nil


let rec mult2 = fun l -> match l with
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let x = Cons(1, Cons(2, Nil)) in
assert(hd (mult2 x) <= 4)
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What about static analysis by abstract interpretation?

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For imperative and object-oriented languages, we have mature static value analyzers.

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- Bautista et al. [2022]: domain for non recursive algebraic values
- Jhala et al. [2011]: HMC, translation into an imperative language

Domains for algebraic data

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» ((y.Cons.0: $\bot$, y.Cons.1: $\bot$), {Nil})

```
let z = Cons(4, x)
```

» ((z.Cons.0: [1, 4], z.Cons.1: {Nil, Cons}), {Cons})

Algebraic Data Types

```
type t =
    | C1 of t1,1 * ... * t1,n_1
    | ...
    | Cm of tm,1 * ... * tn,n_m
```

We choose as an abstract domain:

- We summarize non-recursive field i, j accessible from x : t in one variable $x.i.j$
- We summarize each recursive field by the set of constructors accessible from it.
- We keep track of x's constructor.

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$$\mathcal{D}_{\mathtt{t}} = \prod_{\substack{1 \leq i \leq n \\ 1 \leq j \leq n_i}} \mathcal{D}_{i,j}^{\perp} \times \mathcal{P}(\mathbb{C})$$

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Transfer function: pattern-matching

```
match e with | p1 -> e1 | ... | pn -> en
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We proceed iteratively:

- We evaluate e_i in an over-approximation of environments where e and p_i match.
- We remove p_i pattern and evaluate the result matching in one such that they can't.
- We join the results.

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This method is *flow sensitive* and able to handle `when` clauses.

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match Cons(1, Nil) with
 | Cons(h,q) -> h
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- `Cons`$(1,$ `Nil`$)$ and `Cons`(h, q) match in environments where $h = 1$. Then h evaluates to 1.

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- Then the result is 1.

Functions

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Here, we instantiate $x : (x.0, x.1)$ by $(42, 12)$ so we get $x.0 + x.1 = 42 + 12 = 54$.

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- We ensure convergence in finite time by widening.

The analysis on our example

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- The result is $0 \cup_{\mathbb{Z}} l.\texttt{Cons}.0$

We can summarize the function $\texttt{hd} : l \to 0 \cup_{\mathbb{Z}} l.\texttt{Cons}.0$.

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   - With $l_{cons} = \{\texttt{Nil}\}$, we get $((r.\texttt{Cons}.0 : \bot, \bot), \{\texttt{Nil}\})$
   - By join, we have : $((r.\texttt{Cons}.0 : 2 \times l.\texttt{Cons}.0, \bot), \{\texttt{Cons}, \texttt{Nil}\})$

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2. mult2 : $l \rightarrow ((r.\texttt{Cons.0} : 2 \times l.\texttt{Cons.0}, \{\texttt{Cons}, \texttt{Nil}\}), \{\texttt{Cons}, \texttt{Nil}\})$
3. By analyzing the pattern again, we get the same result: this is a fixpoint.

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3. By analyzing the pattern again, we get the same result: this is a fixpoint.

Then $\texttt{mult2} : l \rightarrow ((r.\texttt{Cons.0} : 2 \times l.\texttt{Cons.0}, \{\texttt{Cons}, \texttt{Nil}\}), \{\texttt{Cons}, \texttt{Nil}\})$.

Example 3

```
let x = Cons(1, Cons(2, Nil)) in
assert(hd (mult2 x) <= 4)
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$$x : ((\texttt{x.Cons.0:[1, 2]}, \texttt{x.Cons.1:\{Nil, Cons\}}), \{\texttt{Cons}\})$$

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$$\begin{cases} \text{mult2: } \text{l} \rightarrow ((r.\texttt{Cons}.0 : 2 \times l.\texttt{Cons}.0, \{\texttt{Cons}, \texttt{Nil}\}), \{\texttt{Cons}, \texttt{Nil}\}) \\ \text{x: } (([1,2], \{\text{Cons, Nil}\}), \{\text{Cons}\}) \end{cases}$$

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$$\implies r_1 : \ (r_1.\text{Cons}.0 : [2,4], \{\texttt{Cons}, \texttt{Nil}\}), \{\texttt{Cons}, \texttt{Nil}\})$$

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let x = Cons(1, Cons(2, Nil)) in
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```

$$\begin{cases} \mathsf{hd}:\ \mathsf{l} \to 0 \cup_{\mathbb{Z}} \mathsf{l}.\mathtt{Cons}.0 \\ \mathsf{r}_1 :\ ([2,4], \{\mathtt{Cons}, \mathtt{Nil}\}), \{\mathtt{Cons}, \mathtt{Nil}\}) \end{cases}$$

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```
let x = Cons(1, Cons(2, Nil)) in
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let x = Cons(1, Cons(2, Nil)) in
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- $r : [0, 4]$

✓ The assertion is proved!

Implementation

MOPSA (Modular Open Platform for Static Analysis)

A modular and multi-language open-source platform:

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- Supporting subsets of C and Python

`https://gitlab.com/mopsa/mopsa-analyzer`

OCaml Analysis

We performed the following implementation steps:

- ✓ Injecting OCaml typed AST into MOPSA AST
- ✓ Designing domains for algebraic values and non-recursive functions
- ✓ Implementing transfer functions for all other constructs (let, type declarations, pattern-matching, etc.)

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- ✓ Implementing transfer functions for all other constructs (let, type declarations, pattern-matching, etc.)

It represents about 2000 lines of OCaml, tested on a few dozens of toy programs, and still has limitations:

- ✗ Implementation to complete (recursive functions)
- ✗ Polymorphism, Higher-order
- ✗ Impure features (mutable arrays, references)
- ✗ But also modules, functors...

Experimental results

Program	Lines	Time (s)
list.ml	4	0.003
tree.ml	2	0.005
match.ml	6	0.004
match_alarm.ml	6	0.005
match_error.ml	6	0.004
add.ml	3	0.004

Figure 1: Execution time on a few toy programs

Conclusion

Conclusion

- A static value analysis for a first-order functional language
- Design of a relational domain for algebraic values
- Implementation into MOPSA platform
- Paving the way towards an analyzer for a higher-order functional language

Further works

- Add support for higher order and polymorphism
- Extend to an impure fragment
- Make the implementation scalable!
- Towards higher-order information: length, depth, or even more sophisticated properties (sort, balance)

Thank you for your attention

Polymorphism and higher-order

For polymorphism, we may:

- Analyze the function for each type instance encountered
- Develop equality and inequality domains for polymorphic data

For higher-order, we may:

- Analyze the function for each function summary in argument
- For numeric information, generalize the current analysis (functions and values are just points of numeric domains)

But we would need new methods for structural information on algebraic values.

Impure features

We'd like to support arrays, references and mutable fields.

- Identify impure variables with types and abstract them to $\top$
- Give them as imputs to every functions
- Identify functions where impure variables don't escape to reduce the cost